PERGAMON

International Journal of Heat and Mass Transfer 42 (1999) 4043–4054

International Journal of
HEAT and MASS
TRANSFER

www.elsevier.com/locate/ijhmt

Effect of temporal variation of pressure on vaporization of liquid droplets

W.J. Sheu*, N.C. Liou

Department of Power Mechanical Engineering, National Tsing Hua University, Hsinchu, Taiwan

Received 13 March 1998; received in revised form 4 December 1998

Abstract

Vaporization of a single liquid droplet in an environment with a temporal variation of pressure is investigated numerically. As a typical case, the evaporation rate (lifetime) of water droplets for a process of adiabatic compression is analyzed. The relative sensitivity of system parameters (i.e. the rate of compression, the initial droplet size and temperature, and the initial temperature and relative humidity in the environment) affecting the vaporization of water droplets is systematically discussed. The results for a process of adiabatic compression are compared with those under an isobaric condition. Physically, the relevant results obtained here are still qualitatively valid for other liquid droplets. © 1999 Elsevier Science Ltd. All rights reserved.

---

1. Introduction

Vaporization of liquid droplets in a hot environment was extensively investigated [1–4]. To understand the evaporation rate of liquid droplets is not only significant on the practical side but also of physical interest on the academic side. Most of previous works were concerned with the vaporization of liquid droplets in an isobaric environment at high temperature. With the gas-phase quasi-steadiness and some other assumptions, a well-known d^2-law was derived analytically. This law is the simplest possible model, stating that the square of the droplet diameter decreases linearly with time as the vaporization of liquid droplets proceeds.

In general, the vaporization of liquid droplets frequently occurs in an environment with a temporal variation of pressure. For example, the vaporization of fuel droplets in Diesel engines takes place in the compression stoke. Besides the practical applications, it is of physical interest to understand the behaviors of the vaporization of liquid droplets under the condition of time-dependent pressure. Thus the objective of the present work is to investigate the evaporation rate and the lifetime of liquid droplets in an environment with the pressure as a function of time. In particular, the process of adiabatic compression is considered.

The water droplet is adopted in the present work because the relevant applications are wide and the thermodynamic properties of water are well known. For generality, the gaseous mixture in the environment may be composed of air and water vapor; thereby the environment is either dry or humid. As a typical case, the system undergoing a process of adiabatic compression at constant rate is of interest. The sensitivity of system parameters (the rate of compression, the relative humidity, the size of droplet, the initial droplet temperature and the initial ambient temperature) affecting the vaporization of water droplet is assessed.

---

* Corresponding author. Tel.: +886-3-5715131; fax: +886-3-5722840.

E-mail address: wjsheu@thccy17.nthu.edu.tw (W.J. Sheu)

0017-9310/99/$ - see front matter © 1999 Elsevier Science Ltd. All rights reserved.
PII: S0017-9310(99)00063-0

Nomenclature

c_P	specific heat at constant pressure
d	diameter of droplet
D	mass diffusivity
k	rate of compression
L	latent heat
M	molecular weight
P	pressure
r	radial coordinate
R	universal gas constant
t	time
T	temperature
v	velocity
Y	mass fraction

Greek symbols

Φ	relative humidity
η	transformed coordinate defined in Eq. (18)
θ	temperature of liquid droplet
λ	thermal conductivity coefficient
ξ	transformed coordinate defined in Eq. (18)
ρ	density

Subscripts

0	initial state
a	air
ev	evaporation
g	gas phase
l	liquid phase
s	surface of droplet
w	water vapor
∞	environment

Superscript

$\cdot$	differentiation with respect to t

Although the vaporization of water droplets is investigated, physically the relevant results obtained here can be applied to other liquid droplets in a qualitative way.

2. Formulation

The spatial gradient of pressure is neglected because the Mach number of flow without any externally forced convection is quite small; thereby only the temporal variation of pressure is considered. The other assumptions are made as follows. The law of ideal gas is satisfied. The effect of gravity is not considered. The thermodynamic equilibrium state is reached at the interface of water droplets. The radiative heat transfer, Soret and Dufour effects are neglected. With the above assumptions, the appropriate governing equations for the unsteady vaporization of a single spherically symmetric water droplet in a hot environment are

gas phase:

$$\frac{\partial \rho_\mathrm{g}}{\partial t}+\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\rho_\mathrm{g}v)=0 \tag{1}$$

$$\rho_\mathrm{g}c_{\mathrm{P,g}}\left(\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial r}\right)=\frac{\mathrm{d}P}{\mathrm{d}t}+\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\lambda_\mathrm{g}\frac{\partial T}{\partial r}\right)+\rho_\mathrm{g}D_\mathrm{w}(c_{\mathrm{P,w}}-c_{\mathrm{P,a}})\frac{\partial T}{\partial r}\frac{\partial Y_\mathrm{w}}{\partial r} \tag{2}$$

$$\rho_\mathrm{g}\left(\frac{\partial Y_\mathrm{w}}{\partial t}+v\frac{\partial Y_\mathrm{w}}{\partial r}\right)=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\rho_\mathrm{g}D_\mathrm{w}\frac{\partial Y_\mathrm{w}}{\partial r}\right) \tag{3}$$

$$P=\rho_\mathrm{g}RT\left(\frac{Y_\mathrm{a}}{M_\mathrm{a}}+\frac{Y_\mathrm{w}}{M_\mathrm{w}}\right) \tag{4}$$

$$Y_\mathrm{w}+Y_\mathrm{a}=1 \tag{5}$$

liquid phase:

$$\rho_\mathrm{l}c_{\mathrm{P,l}}\frac{\partial\theta}{\partial t}=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\lambda_\mathrm{l}\frac{\partial\theta}{\partial r}\right). \tag{6}$$

Boundary conditions are

gas phase:

at $r=r_\mathrm{s}$

$$(\rho_\mathrm{l}-\rho_\mathrm{g})\dot{r}_\mathrm{s}=-\rho_\mathrm{g}v \tag{7}$$

$$\lambda_\mathrm{g}\frac{\partial T}{\partial r}-\lambda_\mathrm{l}\frac{\partial\theta}{\partial r}=-\rho_\mathrm{l}\dot{r}_\mathrm{s}L \tag{8}$$

$$\rho_\mathrm{g}D_\mathrm{w}\frac{\partial Y_\mathrm{w}}{\partial r}=\rho_\mathrm{l}\dot{r}_\mathrm{s}(1-Y_\mathrm{w}) \tag{9}$$

$$Y_\mathrm{w}=\left[1+\frac{M_\mathrm{a}}{M_\mathrm{w}}\left(\frac{P}{P_\mathrm{w}}-1\right)\right]^{-1} \tag{10}$$

as $r\to\infty$

$$\partial T/\partial r\to 0 \quad \text{and} \quad \partial Y_i/\partial r\to 0 \quad \text{for} \quad i=\mathrm{a,w} \tag{11}$$

liquid phase:

at $r = 0$

$$\partial\theta/\partial r = 0 \tag{12}$$

at $r = r_s$

$$\theta = T. \tag{13}$$

Initial conditions at $t = 0$ are

$$r_s = r_{s,0},\ v = 0,\ P = P_0,\ T = T_0,\ \theta = \theta_0 \quad \text{and} \quad Y_i = Y_{i,0} \quad \text{for} \quad i = \text{a, w}. \tag{14}$$

The specific heat at constant pressure and the thermal conductivity in the gas phase are estimated by

$$c_{P,g} = c_{P,a} Y_a + c_{P,w} Y_w \tag{15}$$

$$\lambda_g = \lambda_a Y_a + \lambda_w Y_w, \tag{16}$$

respectively. The magnitudes of $c_{P,a}$, $c_{P,w}$, λ_a and λ_w in Eqs. (15) and (16) are functions of temperature. The values of $c_{P,a}$ and $c_{P,w}$ can be found in the Ref. [5]. The properties λ_a and λ_w are estimated according to curve-fit formulas available in the previous work [6]. Physically the mass diffusion coefficient is inversely proportional to pressure even though the thermal conductivity is not a function of pressure [7]. Because of the lack of experimental data, to a good first approximation the dependence of mass diffusion coefficient on pressure is $D_w \sim P^{-1}$. The curve-fit formula for D_w [6] is

$$D_w = 1.0956 \times 10^{-6} P^{-1} T^{1.76}\ \text{cm}^2/\text{s}. \tag{17}$$

The units of pressure and temperature in Eq. (17) are atm and K, respectively. The specific heat ($c_{P,l}$), the thermal conductivity (λ_l) and the density (ρ_l) in the liquid phase depend on the temperature, respectively [8].

The relationship between T_s and P_w is known [9] when the saturated water vapor is assumed at the surface of water droplets. The latent heat for evaporation L equals the difference of specific enthalpy between saturated-water and saturated-vapor states, that is a function of T_s [9].

3. Coordinate transformation

For the problem with moving boundary conditions, the coordinate transformations are introduced as follows

$$\eta(r,t) = \ln[r/r_s(t)] \quad \text{and} \quad \xi(r,t) = r/r_s(t) \tag{18}$$

for the gas and the liquid phase, respectively.

In terms of the transformed coordinates above, Eqs. (1)–(3) and (6) are

$$\frac{\partial \rho_g}{\partial t} + \frac{1}{r_s e^{3\eta}} \frac{\partial}{\partial \eta}(e^{2\eta}\rho_g v) - \frac{\dot{r}_s}{r_s}\frac{\partial \rho_g}{\partial \eta} = 0 \tag{19}$$

$$\frac{\partial T}{\partial t} - \frac{\lambda_g}{\rho_g c_{P,g} r_s^2 e^{2\eta}}\left\{\frac{\partial^2 T}{\partial \eta^2} + \left[\frac{\rho_g c_{P,g} e^{\eta}}{\lambda_g} r_s(\dot{r}_s e^{\eta} - v) + \frac{1}{\lambda_g}\frac{\partial \lambda_g}{\partial \eta} + 1 - \frac{\rho_g D_w}{\lambda_g}(c_{P,w} - c_{P,a})\frac{\partial Y_w}{\partial \eta}\right]\frac{\partial T}{\partial \eta}\right\} = \frac{dP/dt}{\rho_g c_{P,g}} \tag{20}$$

$$\frac{\partial Y_w}{\partial t} - \frac{D_w}{r_s^2 e^{2\eta}}\left\{\frac{\partial^2 Y_w}{\partial \eta^2} + \left[\frac{e^{\eta} r_s}{D_w}(\dot{r}_s e^{\eta} - v) + \frac{1}{\rho_g D_w}\frac{\partial(\rho_g D_w)}{\partial \eta} + 1\right]\frac{\partial Y_w}{\partial \eta}\right\} = 0 \tag{21}$$

$$\frac{\partial \theta}{\partial t} - \frac{\lambda_l}{\rho_l c_{P,l} r_s^2}\left[\frac{\partial^2 \theta}{\partial \xi^2} + \left(\frac{\rho_l c_{P,l}}{\lambda_l} r_s \dot{r}_s \xi + \frac{2}{\xi} + \frac{1}{\lambda_l}\frac{\partial \lambda_l}{\partial \xi}\right)\frac{\partial \theta}{\partial \xi}\right] = 0. \tag{22}$$

Boundary conditions (8), (9), (11) and (12) become

at $\eta = 0$

$$\lambda_g \frac{\partial T}{\partial \eta} - \lambda_l \frac{\partial \theta}{\partial \xi} = -\rho_l r_s \dot{r}_s L \tag{23}$$

$$\frac{\partial Y_w}{\partial \eta} = \frac{\rho_l r_s \dot{r}_s}{\rho_g D_w}(1 - Y_w) \tag{24}$$

as $\eta \longrightarrow \infty$

$$\partial T/\partial \eta \longrightarrow 0 \quad \text{and} \quad \partial Y_i/\delta\eta \longrightarrow 0 \quad \text{for } i = \text{a,w} \tag{25}$$

at $\xi = 0$

$$\partial\theta/\partial\xi = 0. \tag{26}$$

The other equations and boundary conditions remain the same as those in terms of the original coordinates.

4. Numerical method

The time derivatives of Eqs. (19)–(22) are discretized according to the first-order accurate backward difference scheme. The weighting function method with the

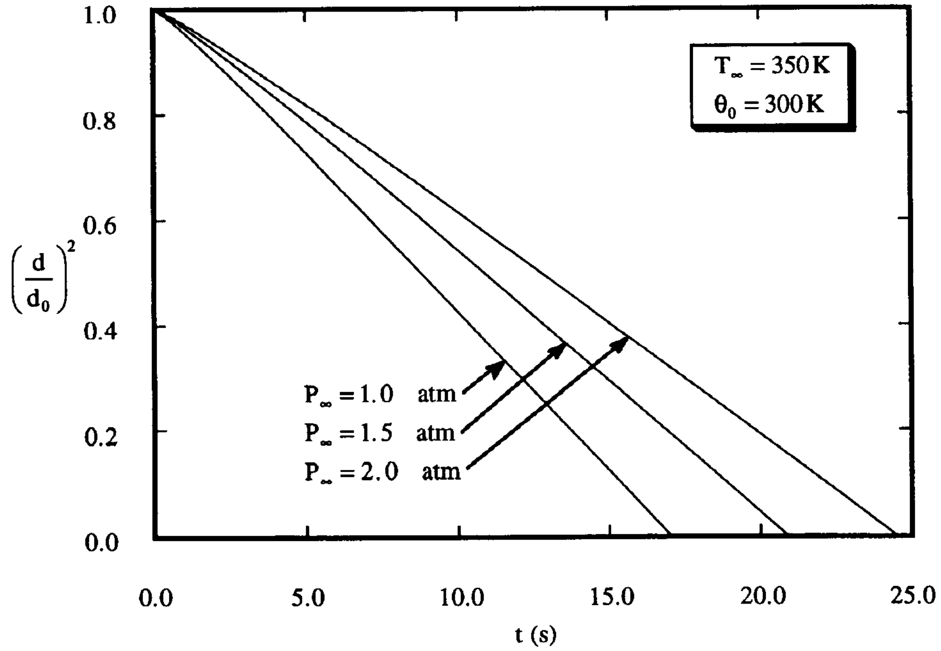

Fig. 1. Plot of $(d/d_0)^2$ vs time for some selected ambient pressures P_∞ ($T_\infty = 350$ K, $\theta_0 = 300$ K, $d_0 = 0.02$ cm and $\Phi_\infty = 0$).

truncated error $O(\Delta\eta^2)$ is applied in the discretization of spatial derivatives [10]. The uniform grid size is adopted in our calculations. The numbers of grid points are 1001 and 51 in the gas and the liquid phase, respectively. The infinite domain $0 \le \eta < \infty$ in the gas phase is truncated in such a way that the location of the boundary does not affect the computed results. The truncated domain corresponds to $0 \le \eta \le 6.0$ $(1 \le r/r_{s,0} \le 403)$.

Initially a value of $\dot{r}_s$ is guessed at each time step. The continuity Eq. (19) is integrated from 0 to η to obtain the distribution of flow velocity (v) at each time step. This integral equation is treated numerically by the trapezoidal method. The energy and species Eqs. (20)–(22) are solved by the weighting function method. Boundary condition (24) is used to modify the magnitude of $\dot{r}_s$ as a new guessed value. After a number of iterations are performed, the correct solution of $\dot{r}_s$ is obtained when the maximum relative errors of all variables less than 5×10^{-4}.

5. Results and discussion

The investigation of the evaporation rate of water droplets under various system conditions is a main objective in the present work. The instantaneous evaporation rate of water droplets is evaluated according to the results of $(d/d_0)^2$ vs time. The lifetime of water droplets is readily defined according to the curves of $(d/d_0)^2$ vs time. The system parameters of interest are the initial size of water droplets, the initial temperature of water droplets, the rate of adiabatic compression, and the temperature and the relative humidity in the environment.

Before the vaporization of water droplets under the condition of adiabatic compression is investigated, the evaporation rate (lifetime) of water droplets in an isobaric environment is analyzed first. In particular, the system becomes isobaric if the rate of compression approaches zero. Physically the isobaric case can be viewed as a special one for a process of compression with an infinitesimal compression rate. Two cases are investigated in this work. The first case is the vaporization of water droplets in an isobaric environment. This implies that the ambient pressure is not a function of time. Although the ambient pressure is independent of time, the evaporation rate of water droplets is still influenced by the magnitude of ambient pressures (e.g. 1 atm, 2 atm, etc.). A comparison between the evaporation rate of water droplets in a high-pressure environment and that in a low-pressure environment is made. The second case is the vaporization of water droplets for a process of adiabatic compression. Obviously, the ambient pressure depends on time and the value of ambient pressure increases with time. From a view-

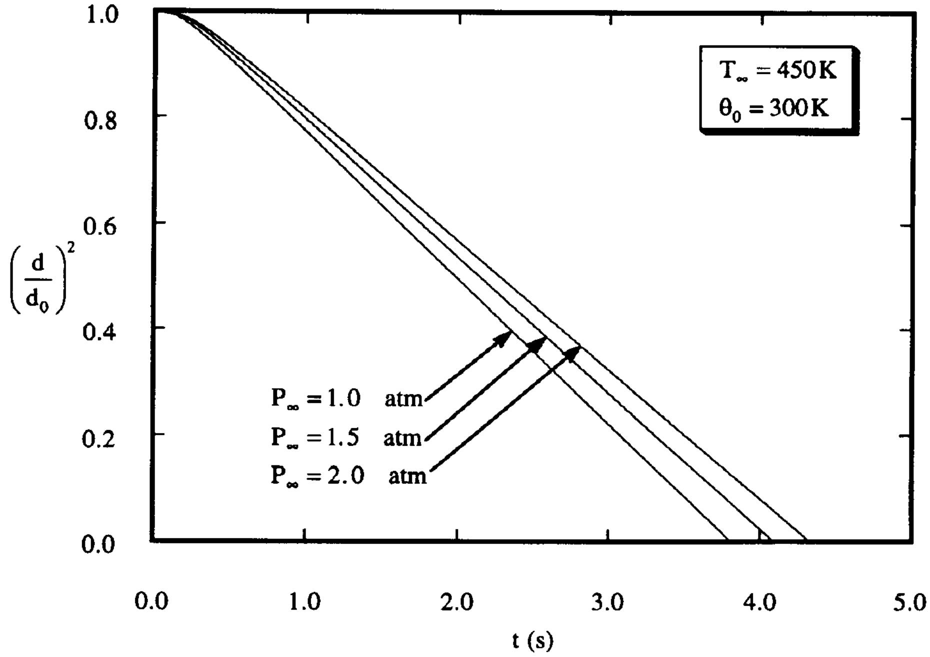

Fig. 2. Plot of $(d/d_0)^2$ vs time for some selected ambient pressures P_∞ ($T_\infty = 450$ K, $\theta_0 = 300$ K, $d_0 = 0.02$ cm and $\Phi_\infty = 0$).

point of thermodynamics, the ambient temperature also increases with time. The rate of compression is a central system parameter for the second case.

5.1. Vaporization of water droplets in an isobaric environment

The ambient pressure (P_∞) is equal to its initial value ($P_{\infty,0}$) during the vaporization of water droplets in an isobaric environment. The effect of ambient pressure on the evaporation rate of water droplets at the same ambient temperature (T_∞) is investigated first. The results of $(d/d_0)^2$ vs time for some selected ambient pressure are illustrated in the Fig. 1 ($T_\infty = 350$ K). Obviously, the d^2-law is valid. Except at the early stage of droplet heating, the evaporation curves of $(d/d_0)^2$ vs time are linear for isobaric cases. According to Fig. 1, the evaporation rate of water droplets at high pressure is smaller than that at low pressure [2,4]. The lifetimes of water droplets are 17.05 s and 24.52 s at $P_\infty = 1$ atm and 2 atm, respectively. The lifetime of water droplets at $P_\infty = 2$ atm is 1.44 times that at $P_\infty = 1$ atm.

The plot of $(d/d_0)^2$ vs time for a great ambient temperature ($T_\infty = 450$ K) is presented in Fig. 2. According to Figs. 1 and 2, the results reveal that the evaporation rate of water droplets increases substantially with T_∞. For $P_\infty = 1$ atm, the lifetime of water droplets with $d_0 = 0.02$ cm reduces greatly from 17.05 s (Fig. 1) to 3.79 s (Fig. 2) as the ambient temperature increases from 350 K to 450 K. According to Fig. 2, three evaporation curves are quite close. As compared with the results in Fig. 1, the evaporation rate (lifetime) of water droplets for great ambient temperature is insensitive to the variation of the ambient pressure. As shown in Fig. 2, the lifetime of water droplets for $P_\infty = 2$ atm is just 1.14 times that for $P_\infty = 1$ atm.

For the investigation of the influence of the initial temperature of water droplets on the vaporization, three distinct values of θ_0, i.e. 285 K, 300 K (Fig. 1) and 330 K, are adopted here. As a typical case, the results of $(d/d_0)^2$ vs time for $\theta_0 = 330$ K are shown in Fig. 3. According to Fig. 1 ($\theta_0 = 300$ K) and 3 ($\theta_0 = 330$ K), the effect of θ_0 on curves of $(d/d_0)^2$ vs time is quite small. The same result is observed for $\theta_0 = 285$ K. The numerical solutions indicate that the lifetime of water droplets for $P_\infty = 1$ atm and $T_\infty = 350$ K varies only from 17.23 s to 16.14 s when the magnitude of θ_0 increases from 285 K to 330 K. As compared with the results of ambient temperature, the influence of the initial temperature of water droplets on the evaporation rate (lifetime) is relatively small.

The magnitude of $(d/d_0)^2$ as a function of time for varied initial size of water droplets is presented in Fig. 4. Obviously, the absolute value of the slope for small size is greater than that for large size. According to the d^2-law, the lifetime of water droplets (t_{ev}) is a function of d_0, i.e. $t_{\text{ev}} \sim d_0^2$. This theoretical prediction is in

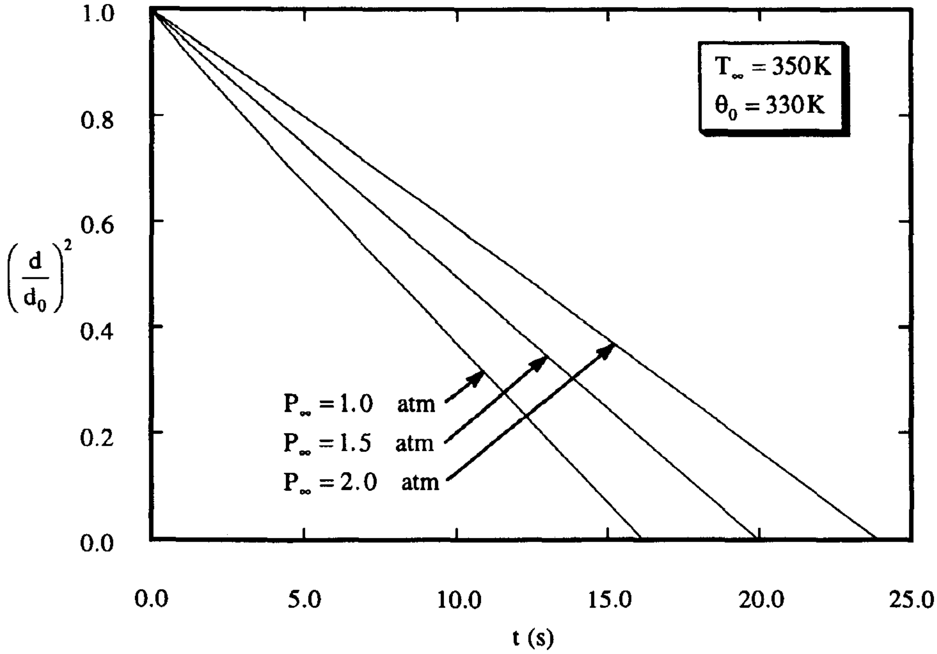

Fig. 3. Plot of $(d/d_0)^2$ vs time for some selected ambient pressures P_∞ ($T_\infty = 350$ K, $\theta_0 = 330$ K, $d_0 = 0.02$ cm and $\Phi_\infty = 0$).

good qualitative agreement with the numerical results obtained here. The lifetime of water droplets increases from 4.35 s to 17.05 s when the droplet size varies from 0.01 cm to 0.02 cm.

The results of $(d/d_0)^2$ vs time for varied relative humidity in the environment Φ_∞ are shown in Figs. 5 ($T_\infty = 350$ K) and 6 ($T_\infty = 450$ K). According to these figures, the lifetime of water droplets increases with the

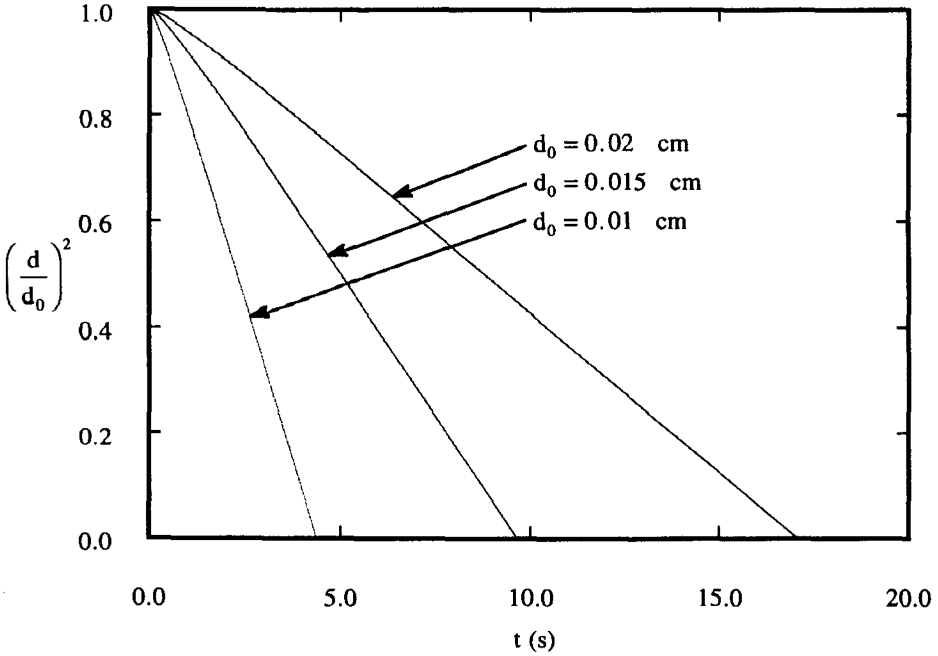

Fig. 4. Plot of $(d/d_0)^2$ vs time for varied d_0 ($T_\infty = 350$ K, $\theta_0 = 300$ K, $P_\infty = 1$ atm and $\Phi_\infty = 0$).

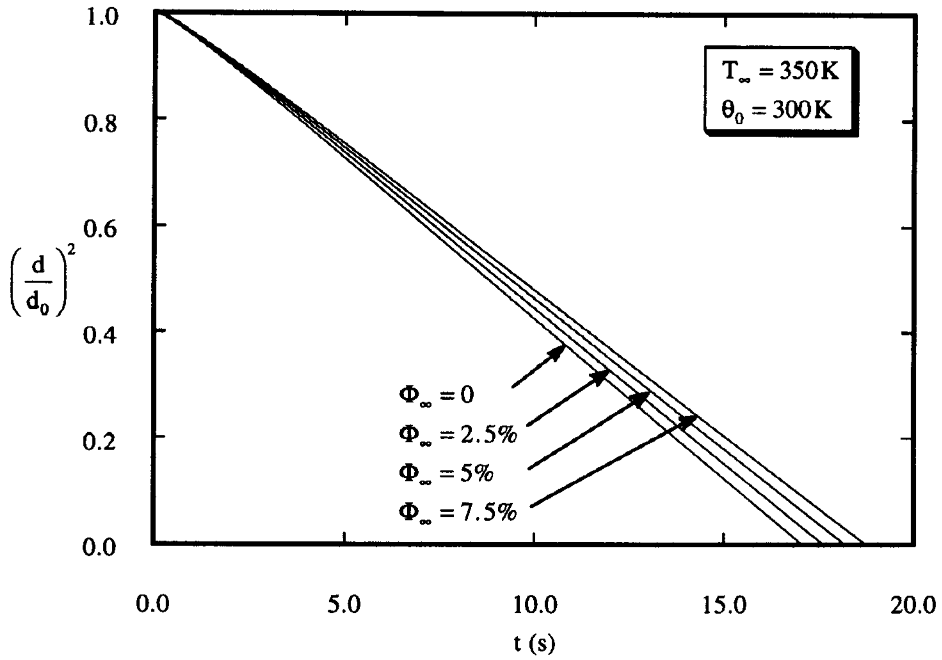

Fig. 5. Plot of $(d/d_0)^2$ vs time for varied Φ_∞ ($T_\infty = 350$ K, $\theta_0 = 300$ K, $d_0 = 0.02$ cm and $P_\infty = 1$ atm).

relative humidity, as expected physically. For $T_\infty = 450$ K (Fig. 6), the condensation apparently occurs on the surface of water droplets initially for $\Phi_\infty > 2.5\%$ when cold water droplets are placed in the high-temperature environment with humidity. According to Figs. 5 and 6, the relative variation of evaporation rate (lifetime) with Φ_∞ increases with the ambient temperature. Therefore, the effect of humidity on the vaporization of water droplets becomes important in an environment at great temperature.

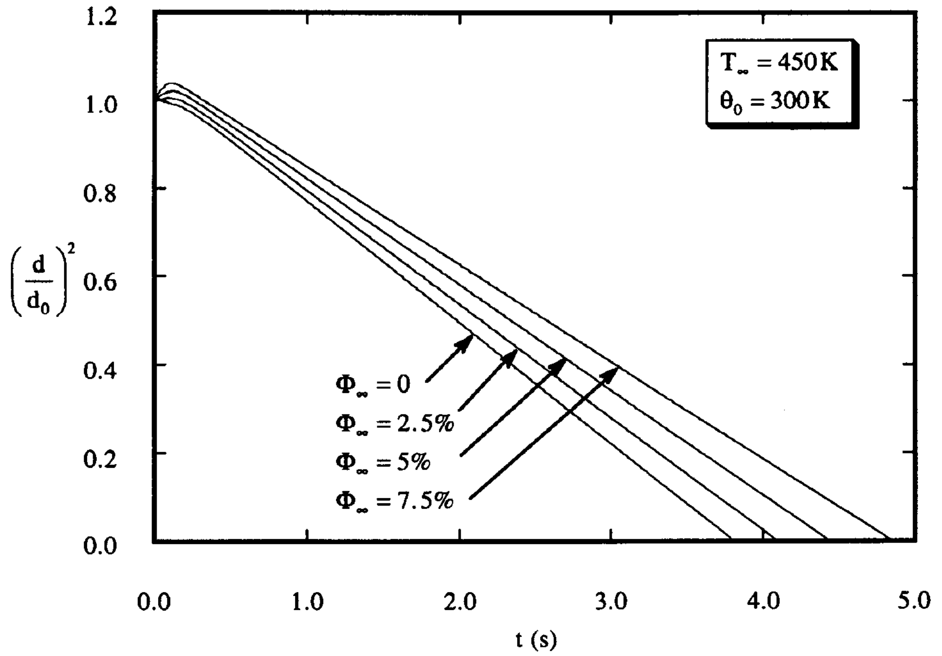

Fig. 6. Plot of $(d/d_0)^2$ vs time for varied Φ_∞ ($T_\infty = 450$ K, $\theta_0 = 300$ K, $d_0 = 0.02$ cm and $P_\infty = 1$ atm).

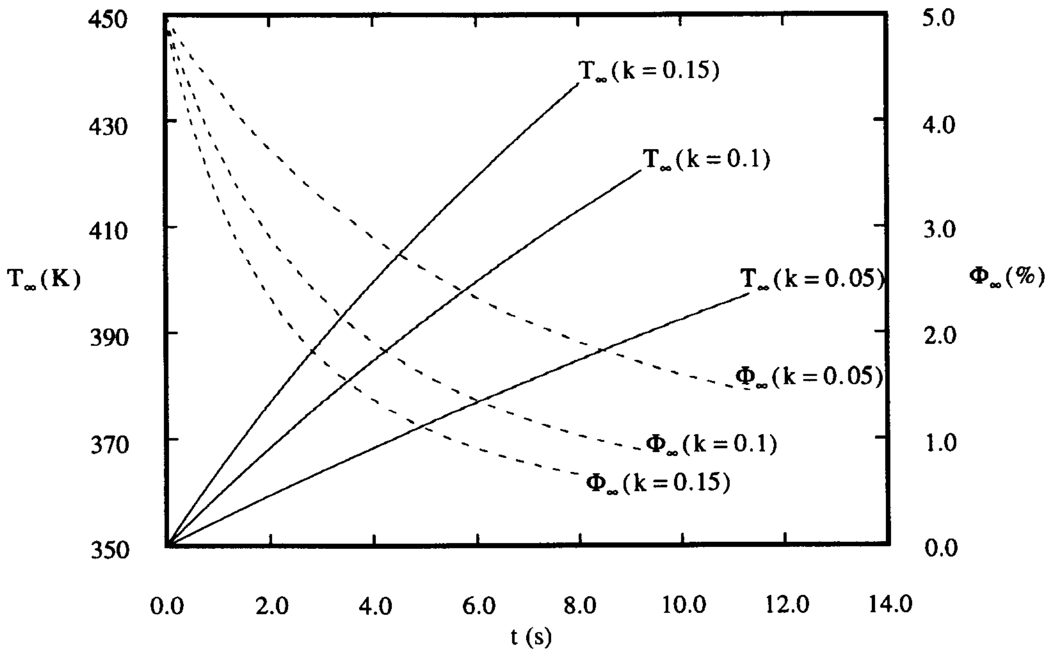

Fig. 7. Results of T_∞ and Φ_∞ vs time for varied k ($=\mathrm{d}P/\mathrm{d}t$) ($T_{\infty,0}=350$ K, $P_{\infty,0}=1$ atm, $\Phi_{\infty,0}=5\%$, $d_0=0.02$ cm and $\theta_0=300$ K).

5.2. Vaporization of water droplets for a process of adiabatic compression

For simplicity, the linear function of pressure vs time (i.e. $P=P_{\infty,0}+kt$, where k is a constant) is considered in the present work. According to this linear function, physically this system undergoes a process of adiabatic compression at constant rate. The magnitude

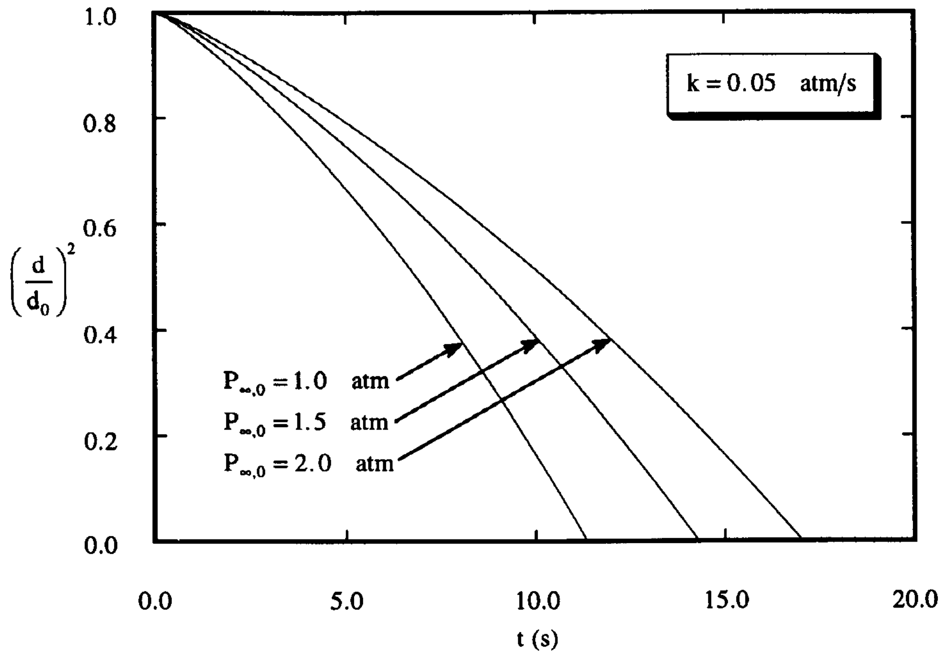

Fig. 8. Plot of $(d/d_0)^2$ vs time with k = 0.05 atm/s for $P_{\infty,0}=1.0$, 1.5 and 2.0 atm ($T_{\infty,0}=350$ K, $\Phi_{\infty,0}=0$, $d_0=0.02$ cm and $\theta_0=300$ K).

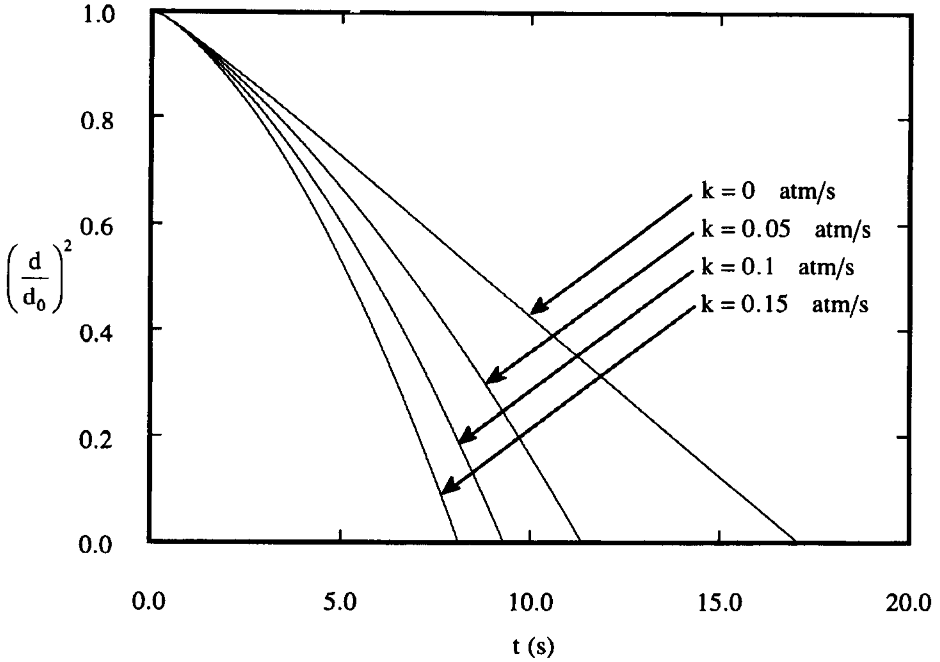

Fig. 9. Plot of $(d/d_0)^2$ vs time for varied k ($=\mathrm{d}P/\mathrm{d}t$) ($T_{\infty,0}=350$ K, $P_{\infty,0}=1$ atm, $\Phi_{\infty,0}=0$, $d_0=0.02$ cm and $\theta_0=300$ K).

of k ($=\mathrm{d}P/\mathrm{d}t$) is viewed as the rate of compression. Contrary to conditions for the isobaric case ($k = 0$), both the ambient temperature T_∞ and the relative humidity Φ_∞ depend on time for a process of adiabatic compression. The results of T_∞ and Φ_∞ vs k are illustrated in Fig. 7 ($T_{\infty,0}=350$ K and $\Phi_{\infty,0}$). For $k \neq 0$, the ambient temperature increases with time, as expected physically. However, the relative humidity decreases with time during the process of adiabatic compression due to an increase in the ambient temperature. These effects become great as the rate of compression increases.

The magnitude of $(d/d_0)^2$ as a function of time with $k = 0.05$ atm/s for varied $P_{\infty,0}$ is shown in Fig. 8. As compared with the results in Fig. 1 ($k = 0$ atm/s), the curves of $(d/d_0)^2$ vs time in Fig. 8 are no longer linear for $k \neq 0$. Obviously, the d^2-law is not valid for a process of adiabatic compression. According to Figs. 1 and 8, the lifetimes of water droplets for $k=0.05$ atm/s are invariably smaller than those for $k = 0$ atm/s (isobaric condition). Physically, the additional energy is provided by a process of adiabatic compression. This fact implies that the evaporation rate of water droplets will be enhanced if a process of adiabatic compression is imposed.

The plot of $(d/d_0)^2$ vs time for $k = 0$, 0.05, 0.1 and 0.15 atm/s is presented in Fig. 9. The case for $k = 0$ is the system under an isobaric condition. The lifetime of water droplets for $k = 0$ is 17.05 s. The evaporation rate of water droplets increases with the rate of compression because the increasing rate of ambient temperature becomes great when the rate of compression increases (Fig. 7). According to Fig. 9, the lifetime of water droplets for $k = 0.05$ atm/s decreases from 17.05 s ($k = 0$) to 11.34 s. The evaporation rate of water droplets increases greatly when a process of adiabatic compression at small rate is imposed. However, the lifetime of water droplets decreases only from 9.28 s to 8.08 s when the rate of compression increases from 0.1 atm/s to 0.15 atm/s. Consequently, the evaporation rate (lifetime) of water droplets is insensitive to a further increase in the rate of compression for a great rate of compression.

It is interesting to investigate the evaporation rate of droplets during a short initial period because the linearity of the d^2-law is absent even in an isobaric environment. As is well known, the non-linear behavior of evaporation rate in an isobaric environment is caused mainly by droplet heating. Physically the droplet size hardly changes at the early stage of droplet heating. Besides the droplet heating, the characteristics of this non-linearity are further modified by a process of adiabatic compression. The evaporation rate is enhanced throughout the lifetime of droplets for a process of adiabatic compression such that the absolute value of the slope of evaporation curves increases with the rate of compression as the droplet heating starts.

The lifetime of water droplets (t_{ev}) as a function of k for varied $T_{\infty,0}$ is shown in Fig. 10. According to this figure, the effect of adiabatic compression on the life-

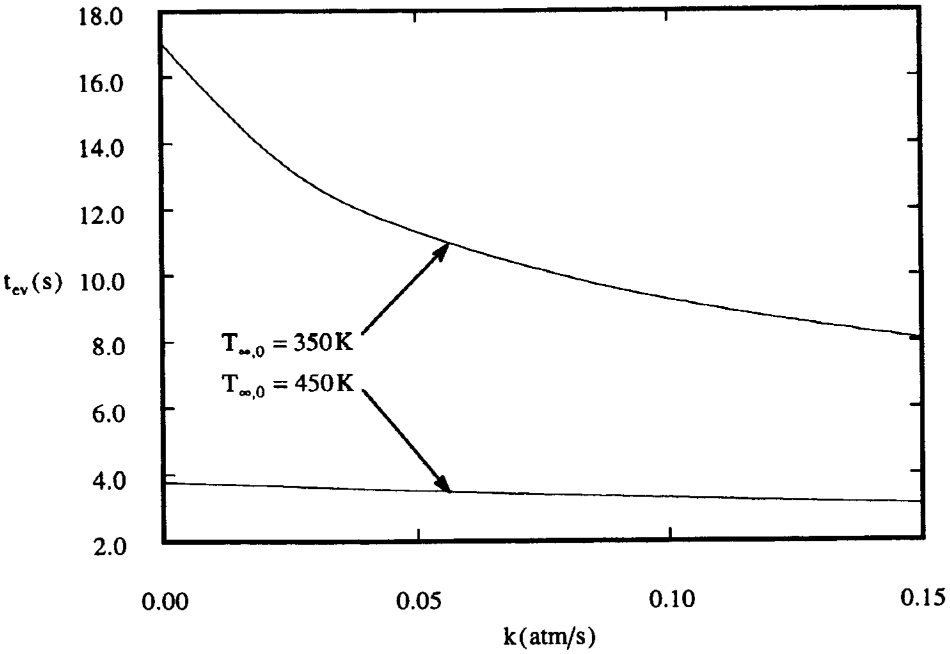

Fig. 10. Plot of t_{ev} vs k for varied $T_{\infty,0}$ ($P_{\infty,0}=1$ atm, $\Phi_{\infty,0}=0$ $d_0=0.02$ cm and $\theta_0=300$ K).

time of water droplets increases with decreasing initial ambient temperature. For $T_{\infty,0}=350$ K, the lifetime decreases substantially from 17.05 s to 8.08 s as k increases from 0.0 to 0.15 atm/s. However, the lifetime of water droplets for great $T_{\infty,0}$ (450 K in Fig. 10) almost remains the same for the entire range of k. The above results reveal that the influence of adiabatic compression on the evaporation rate (lifetime) of water droplets is significant for small $T_{\infty,0}$.

The plot of t_{ev} vs k for varied d_0 is provided in Fig.

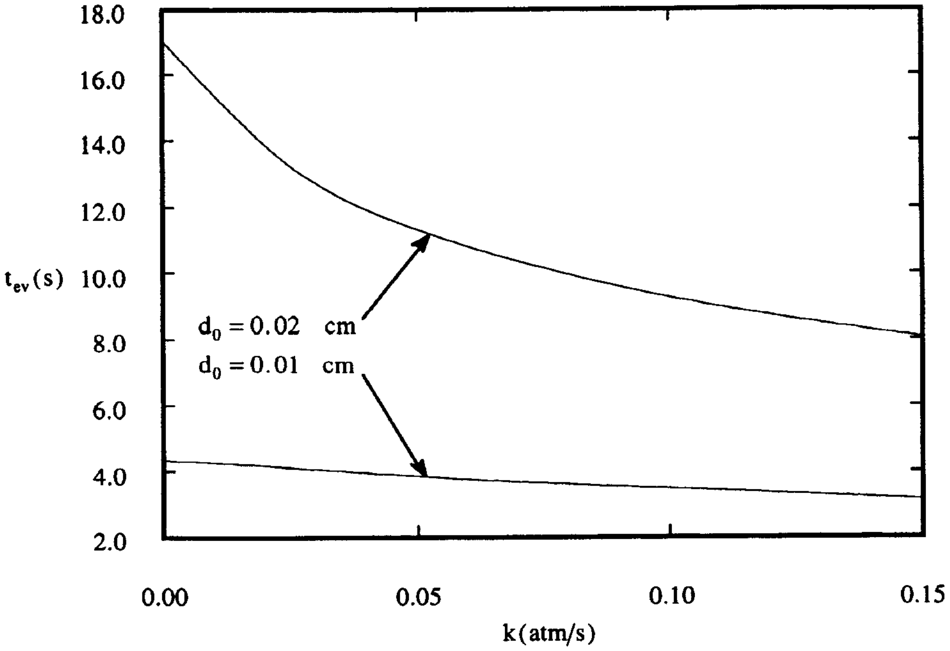

Fig. 11. Plot of t_{ev} vs k for varied d_0 ($T_{\infty,0}=350$ K, $P_{\infty,0}=1$ atm, $\Phi_{\infty,0}=0$ and $\theta_0=300$ K).

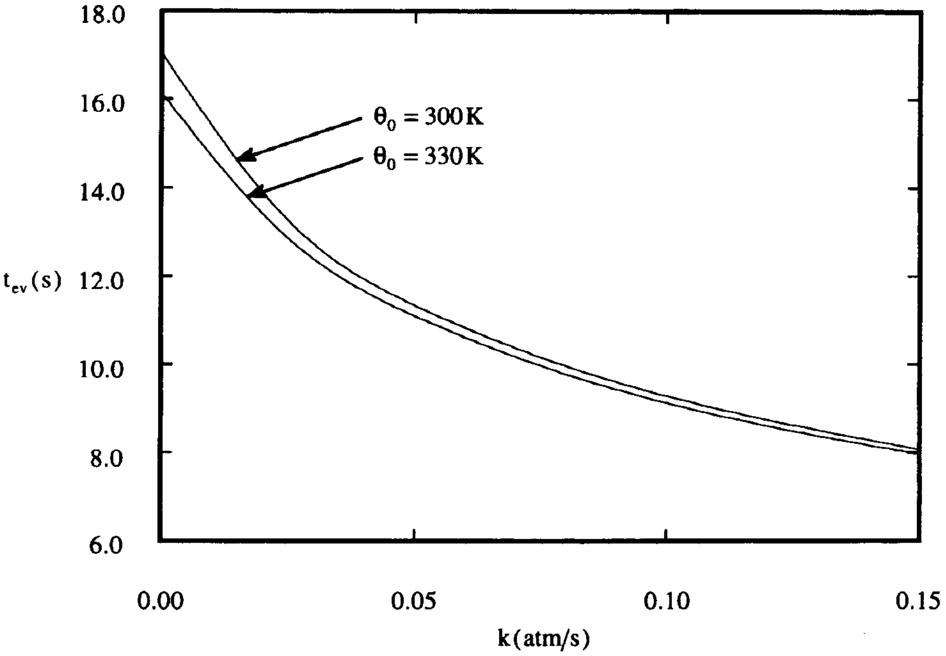

Fig. 12. Plot of t_{ev} vs k for varied θ_0 ($T_{\infty,0}=350$ K, $P_{\infty,0}=1$ atm, $\Phi_{\infty,0}=0$ and $d_0=0.02$ cm).

11. The results reveal that the influence of adiabatic compression on t_{ev} for $d_0=0.02$ cm is greater than that for $d_0=0.01$ cm. The lifetime is sensitive to the variation of the rate of compression for large water droplets ($d_0=0.02$ cm). However, as the magnitude of k increases the modification of lifetime is not obvious for small water droplets ($d_0=0.01$ cm). Thus the effect of adiabatic compression on the vaporization of water droplets increases with the droplet size.

The dependence of t_{ev} on k for varied θ_0 is presented

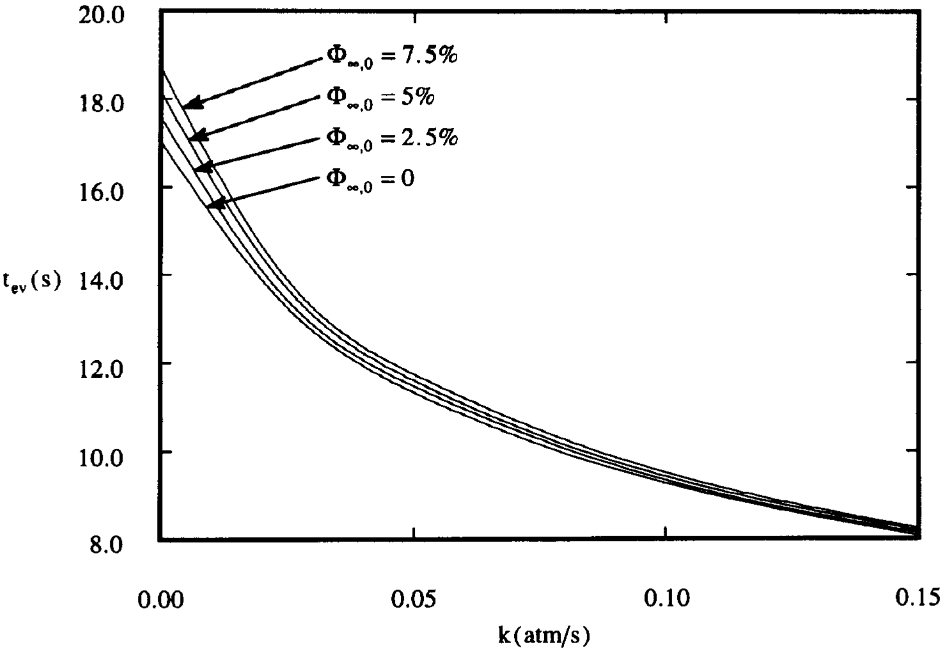

Fig. 13. Plot of t_{ev} vs k for varied $\Phi_{\infty,0}$ ($T_{\infty,0}=350$ K, $P_{\infty,0}=1$ atm, $d_0=0.02$ cm and $\theta_0=300$ K).

in Fig. 12. The results reveal that effect of θ_0 on t_{ev} decreases gradually with the rate of compression according to this figure. The effect of initial relative humidity $\Phi_{\infty,0}$ on t_{ev} is illustrated in Fig. 13. The influence of $\Phi_{\infty,0}$ on the vaporization of water droplets is, relatively, great when the rate of compression is small. According to Figs. 12 and 13, these facts imply that the effects of θ_0 and $\Phi_{\infty,0}$ on t_{ev} are negligible for great k.

6. Conclusions

The vaporization of water droplets for a process of adiabatic compression is investigated numerically in the present work. The following results are obtained.

1. Vaporization of water droplets in an isobaric environment.

   In the limit $k \to 0$, the problem of interest becomes isobaric. The ambient pressure is time-independent under this condition. The evaporation rate of water droplets decreases with the ambient pressure; thereby the lifetime of water droplets increases with the ambient pressure. The effect of ambient pressure on the vaporization of droplets increases gradually with decreasing the ambient temperature. Physically, the evaporation rate (lifetime) of water droplets in an environment at small temperature is influenced greatly by the ambient pressure. As compared with the effect of ambient temperature, the effect of the initial temperature of water droplets on the evaporation rate is relatively small. The evaporation rate of water droplets decreases with the relative humidity in the environment. The effect of relative humidity on the vaporization of water droplets increases with the ambient temperature.
2. Vaporization of water droplets for a process of adiabatic compression.

   A process of adiabatic compression is considered here. The d^2-law is not applicable in this case. As a process of adiabatic compression starts, the temperature and the relative humidity in the environment increases and decreases with the ambient pressure, respectively. The evaporation rate of water droplets for a process of adiabatic compression is invariably greater than those in an isobaric environment. The evaporation rate of water droplets increases with the rate of compression k; thereby the lifetime of water droplets decreases with k. With the same increase in the rate of compression, the reduction in lifetimes of water droplets for small k is more pronounced than that for great k. The effect of k on the vaporization of water droplets increases as the initial ambient temperature decreases. The influence of k on the vaporization of water droplets increases with the initial droplet size. The effects of the initial temperature of water droplets θ_0 and the initial relative humidity $\Phi_{\infty,0}$ on the vaporization of water droplets decreases gradually with the rate of compression. The effects of θ_0 and $\Phi_{\infty,0}$ are important only for small k.

The physical results above may be applied to other liquid droplets if their thermodynamic properties are qualitatively the same as those of water droplets.

References

[1] A. Williams, Combustion of droplets of liquid fuels: a review, Combust. Flame 21 (1973) 1–31.

[2] G.L. Hubbard, V.E. Denny, A.F. Mills, Droplet evaporation: effect of transients and variable properties, Int. J. Heat Mass Transfer 18 (1975) 1003–1008.

[3] C.K. Law, Recent advances in droplet vaporization and combustion, Prog. Energy Combust. Sci. 8 (1982) 171–201.

[4] H. Jia, G. Gogos, High pressure droplet vaporization; effects of liquid-phase gas solubility, Int. J. Heat Mass Transfer 36 (1993) 4419–4431.

[5] R.C. Reid, J.M. Prausnitz, B.E. Poling, The Properties of Gases & Liquids, 4th ed., McGraw-Hill, New York, 1988, p. 656.

[6] W.J. Sheu, M.C. Lin, Upstream interactions between premixed flames, Combust. Sci. Tech. 110–111 (1995) 1–17.

[7] R.A. Strehlow, Combustion Fundamentals, McGraw-Hill, New York, 1985, p. 67.

[8] J.P. Holman, Heat Transfer, 4th ed., McGraw-Hill, New York, 1976, p. 507.

[9] J.P. Holman, in: Thermodynamics, 2nd ed., McGraw-Hill, New York, 1975, p. 551.

[10] S.L. Lee, Weighting function scheme and its application on multidimensional conservation equations, Int. J. Heat Mass Transfer 32 (1989) 2065–2073.